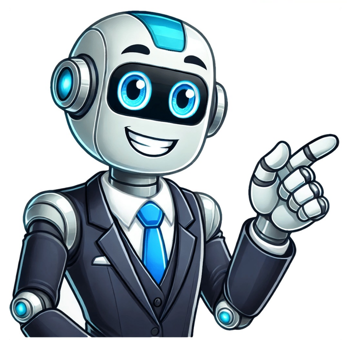

The addition x + a on the number line. All numbers greater than x and less than x + a fall within that open interval.In mathematics, a (real) interval is a set of real numbers that contains all real numbers lying between any two numbers of the set. For example, the set of numbers x satisfying 0 x 1 is an interval which contains 0, 1, and all numbers in between. Other examples of intervals are the set of numbers such that 0 < x < 1, the set of all real numbers R {\displaystyle \mathbb {R} } , the set of nonnegative real numbers, the set of positive real numbers, the empty set, and any singleton (set of one element).Real intervals play an important role in the theory of integration, because they are the simplest sets whose "size" (or "measure" or "length") is easy to define. The concept of measure can then be extended to more complicated sets of real numbers, leading to the Borel measure and eventually to the Lebesgue measure.Intervals are central to interval arithmetic, a general numerical computing technique that automatically provides guaranteed enclosures for arbitrary formulas, even in the presence of uncertainties, mathematical approximations, and arithmetic roundoff.Intervals are likewise defined on an arbitrary totally ordered set, such as integers or rational numbers. The notation of integer intervals is considered in the special section below. TerminologyAn open interval does not include its endpoints, and is indicated with parentheses. For example, (0,1) means greater than 0 and less than 1. This means (0,1) = {x | 0 < x < 1}.A closed interval is an interval which includes all its limit points, and is denoted with square brackets. For example, [0,1] means greater than or equal to 0 and less than or equal to 1.A half-open interval includes only one of its endpoints, and is denoted by mixing the notations for open and closed intervals. For example, (0,1] means greater than 0 and less than or equal to 1, while [0,1) means greater than or equal to 0 and less than 1.A degenerate interval is any set consisting of a single real number (i.e., an interval of the form [a,a]). Some authors include the empty set in this definition. A real interval that is neither empty nor degenerate is said to be proper, and has infinitely many elements.An interval is said to be left-bounded or right-bounded, if there is some real number that is, respectively, smaller than or larger than all its elements. An interval is said to be bounded, if it is both left- and right-bounded; and is said to be unbounded otherwise. Intervals that are bounded at only one end are said to be half-bounded. The empty set is bounded, and the set of all reals is the only interval that is unbounded at both ends. Bounded intervals are also commonly known as finite intervals.Bounded intervals are bounded sets, in the sense that their diameter (which is equal to the absolute difference between the endpoints) is finite. The diameter may be called the length, width, measure, range, or size of the interval. The size of unbounded intervals is usually defined as +, and the size of the empty interval may be defined as 0 (or left undefined).The centre (midpoint) of bounded interval with endpoints a and b is (a+b)/2, and its radius is the half-length |ab|/2. These concepts are undefined for empty or unbounded intervals.An interval is said to be left-open if and only if it contains no minimum (an element that is smaller than all other elements); right-open if it contains no maximum; and open if it has both properties. The interval [0,1) = {x | 0 x < 1}, for example, is left-closed and right-open. The empty set and the set of all reals are open intervals, while the set of non-negative reals, is a right-open but not left-open interval. The open intervals are open sets of the real line in its standard topology, and form a base of the open sets.An interval is said to be left-closed if it has a minimum element, right-closed if it has a maximum, and simply closed if it has both. These definitions are usually extended to include the empty set and the (left- or right-) unbounded intervals, so that the closed intervals coincide with closed sets in that topology.The interior of an interval I is the largest open interval that is contained in I; it is also the set of points in I which are not endpoints of I. The closure of I is the smallest closed interval that contains I; which is also the set I augmented with its finite endpoints.For any set X of real numbers, the interval enclosure or interval span of X is the unique interval that contains X, and does not properly contain any other interval that also contains X.An interval I is subinterval of interval J if I is a subset of J. An interval I is a proper subinterval of J if I is a proper subset of J.Note on conflicting terminologyThe terms segment and interval have been employed in the literature in two essentially opposite ways, resulting in ambiguity when these terms are used. The Encyclopedia of Mathematics defines interval (without a qualifier) to exclude both endpoints (i.e., open interval) and segment to include both endpoints (i.e., closed interval), while Rudin's Principles of Mathematical Analysis calls sets of the form [a, b] intervals and sets of the form (a, b) segments throughout. These terms tend to appear in older works; modern texts increasingly favor the term interval (qualified by open, closed, or half-open), regardless of whether endpoints are included.Notations for intervalsThe interval of numbers between a and b, including a and b, is often denoted [a,b]. The two numbers are called the endpoints of the interval. In countries where numbers are written with a decimal comma, a semicolon may be used as a separator to avoid ambiguity.Including or excluding endpointsTo indicate that one of the endpoints is to be excluded from the set, the corresponding square bracket can be either replaced with a parenthesis, or reversed. Both notations are described in International standard ISO 31-11. Thus, in set builder notation, (a , b) =] a , b [= { x R a < x < b } , [a , b) = [a , b [= { x R a x < b } , (a , b] =] a , b] = { x R a < x b } , [a , b] = [a , b] = { x R a x b } . {\displaystyle {\begin{aligned}{\color {Maroon}(}a,b{\color {Maroon})}={\mathopen {\color {Maroon}]}}a,b{\mathclose {\color {Maroon}[}}&=\{x\in \mathbb {R} \mid a{\color {Maroon}{}